

# **MJC Physics – Semester II**

## **Unit-1 : Oscillations**

**Topic: Free, Damped and Forced  
Oscillations**

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# 1. Introduction

Oscillatory motion is a type of periodic motion in which a system moves to and fro about a mean (equilibrium) position. Depending on the presence of resistive forces and external driving forces, oscillations are classified as:

1. Free oscillations
  2. Damped oscillations
  3. Forced oscillations
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# 2. Free Oscillations

## Definition

Free oscillations are those oscillations in which a system, once displaced from its equilibrium position, oscillates under the action of its own restoring force only, without any external force or resistive force acting on it.

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## Example

- Simple pendulum oscillating in vacuum
  - Mass–spring system without friction
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# Equation of Motion

Consider a particle of mass  $m$  attached to a spring of force constant  $k$ .

Restoring force,

$$F = -kx$$

Using Newton's second law,

$$m \frac{d^2 x}{dt^2} = -kx$$

or,

$$\frac{d^2 x}{dt^2} + \omega^2 x = 0$$

where,

$$\omega = \sqrt{\frac{k}{m}}$$

# Solution

The solution of the above differential equation is:

$$x = A \sin(\omega t + \phi)$$

where

$A$  = amplitude

$\omega$  = angular frequency

$\phi$  = phase constant

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# Time Period

$$T = \frac{2\pi}{\omega}$$

# Characteristics of Free Oscillations

- Amplitude remains constant
  - Frequency depends only on system parameters
  - Energy remains constant
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## 3. Damped Oscillations

### Definition

Damped oscillations are those oscillations in which the amplitude decreases gradually with time due to the presence of resistive forces such as friction or air resistance.

# Damping Force

Damping force is proportional to velocity:

$$F_d = -r \frac{dx}{dt}$$

where  $r$  is the damping constant.

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## Equation of Motion

$$m \frac{d^2 x}{dt^2} + r \frac{dx}{dt} + kx = 0$$

Dividing by  $m$ ,

$$\frac{d^2 x}{dt^2} + 2\beta \frac{dx}{dt} + \omega_0^2 x = 0$$

where,

$$\beta = \frac{r}{2m}, \quad \omega_0 = \sqrt{\frac{k}{m}}$$

## Solution (Underdamped Case)

$$x = Ae^{-\beta t} \sin(\omega t + \phi)$$

where,

$$\omega = \sqrt{\omega_0^2 - \beta^2}$$

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## Energy in Damped Oscillation

Energy decreases exponentially with time:

$$E = E_0 e^{-2\beta t}$$

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# Types of Damping

1. **Underdamping** – oscillatory motion with decreasing amplitude
  2. **Critical damping** – fastest return to equilibrium without oscillation
  3. **Overdamping** – slow return to equilibrium
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## Characteristics

- Amplitude decreases exponentially
  - Frequency slightly less than natural frequency
  - Energy continuously lost
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# 4. Forced Oscillations

## Definition

Forced oscillations are those oscillations in which a system is acted upon by an external periodic force.

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## External Driving Force

$$F = F_0 \sin(\omega t)$$

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## Equation of Motion

$$m \frac{d^2 x}{dt^2} + r \frac{dx}{dt} + kx = F_0 \sin(\omega t)$$

## Steady State Solution

$$x = A \sin(\omega t - \delta)$$

where amplitude,

$$A = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2}}$$

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## Phase Difference

$$\tan \delta = \frac{2\beta\omega}{\omega_0^2 - \omega^2}$$

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## Resonance

Resonance occurs when the amplitude of forced oscillation becomes maximum.

Resonant frequency,

$$\omega_r = \sqrt{\omega_0^2 - 2\beta^2}$$

## Characteristics

- Amplitude depends on driving frequency
- Phase difference changes with frequency
- Maximum energy absorption at resonance

## 5. Comparison of Oscillations

<b>Feature</b>	<b>Free</b>	<b>Damped</b>
External force	No	No
Amplitude	Constant	Decreasing
Energy	Constant	Decreases
Frequency	Natural	Slightly reduced

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## 6. Applications

- Shock absorbers
- AC circuits
- Mechanical vibrations
- Radio and TV tuning

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## Conclusion

Free, damped, and forced oscillations form the foundation of oscillatory motion in physics. Understanding their mathematical treatment helps in analyzing real physical systems where friction and external forces are always present.